

Mathematics Tutorial Series

Integral Calculus #9

Integration by Substitution

Integration by substitution is really just learning how to change the variable.

Simple example:

$$\int e^{mx} dx$$

Change the variable to u where $u = mx$.

The new variable changes with respect to the old variable at the rate $\frac{du}{dx} = m$.

Write this as

$$du = m dx$$

or

$$dx = \frac{1}{m} du$$

$$\int e^{mx} dx = \int e^u \frac{1}{m} du = \frac{1}{m} e^u + C = \frac{1}{m} e^{mx} + C$$

Example 2

$$\int 2x \cos x^2 dx$$

Let $u = x^2$ so that $du = 2x dx$.

Substitute:

$$\begin{aligned} \int 2x \cos x^2 dx \\ &= \int \cos u du \\ &= \sin u + C \\ &= \sin x^2 + C \end{aligned}$$

Check, using the chain rule

$$(\sin x^2)' = \cos x^2 \cdot 2x$$

Summary

1. Integration by substitution is a change of variable technique.
2. The goal is to make the integral simpler after the substitution.
3. Look for a part of the integrand that also has its derivative in the integrand:
 $u = x^2$ so that $du = 2x dx$.
4. You can always check an integral by differentiation.